Ion-sound Velocity at the Plasma Edge in Fusion-Relevant Plasmas

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Abstract

The plasma-sheath boundary is confirmed to be a surface at which the ion directional velocity towards the wall attains the ion-sound velocity (i.e., the "sonic point" or "Mach-1 point"). Expressing this statement quantitatively is a complex task which has been solved to a satisfactory degree for low temperature plasmas only, where the ion temperature is negligible with respect to the electron population temperature (i.e. "cold-ion" plasmas). In this paper we tackle this problem for "warm-ion" plasmas, in which the ion temperature is arbitrary. The proposed method is perfectly suited for fusion-relevant plasmas.

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1. Introduction

The plasma-sheath boundary in general plasma theory is identified as a surface at which the ion directional velocity u_i towards the wall attains the sonic velocity defined as $c_s = \sqrt{kT_e + \gamma_i kT_i}$ [where k is the Boltzmann constant, $T_{e,i}$ are the electron and ion temperatures respectively, and γ_i is the ion polytropic coefficient function (Kuhn et al., 2006)] becomes supersonic. Both u_i and c_s are quantities which today, in principle, can be predicted and measured, at least in kinetic computational simulations. However, it is an expensive and laborious task to apply such codes for complex plasmas as in fusion devices, so fluid models instead are in widespread use, which, however, still do not involve enough physics. Application of such codes makes it important not only to declare what the plasma-sheath boundary is but, rather, to express *quantitatively* either the ion-sound or the directional ion velocity, or both of them. In particular, in numerical simulations of fusion plasmas, where it is necessary to impose well-defined boundary conditions for employing numerical codes, e.g., SOLPS code (Coster, 2003), it is necessary to obtain plasma parameters behaviors under various physical scenarios of interest in the scrape-off layer (SOL) region in Tokamak devices, and these still are *not known* with sufficient reliability. Defining the ion-sound (and related directional) velocity is a complex task which has been solved to a satisfactory degree for low temperature plasmas only, where, in addition, the ion temperature is negligible with respect to the electron population temperature, so that the precise knowledge of the value of product $\gamma_i kT_i$ is not so critical. However, it has been shown by Kuhn et al. (2006) that even in plasmas in which the ions are born at rest (with zero initial velocities), the ion temperature is not exactly zero, and moreover, that the value of the polytropic coefficient at the boundary might be several times higher than previously supposed in plasma physics. Obviously, in fusion plasmas where the ion and electron temperatures are of the same order of magnitude the relevancy of product $\gamma_i kT_i$ becomes a critical one, primarily because even a particular value for γ_i is not generally known with reliability.

Our approach to solve the problem of ion-sound velocity is based on the extended formulation of the Tonks-Langmuir theory of the plasma arc (Tonks and Langmuir, 1929) as schematically shown in Fig. 1 sketched for the plane geometry. The problem consists of finding a potential profile together with ion velocity distribution, provided the electron density distribution is known, and the mechanisms of ion production and energy gains and losses are well defined. Schematic potential profile $\Phi(x)$ is shown in the case of a negligible ε. This means that in a very thin sheath region the main potential drop $\Phi_s - \Phi_w$ is located, (where Φ_s - the plasma-sheath potential drop as measured with respect to the center of discharge is the point at which a sudden drop of the electric

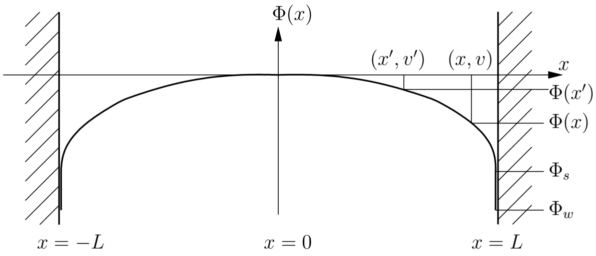

Figure 1: Schematic diagram of the T&L model in one-dimensional (plane) geometry with potential $\Phi(x)$. The plasma center at $x = 0$, walls at $x = \pm L$. Φ_s is the potential of the sheath edge, Φ_w is the wall potential.

field $E \equiv -1/\Psi(\Phi)$ is situated, and Φ_w is the wall potential to be found self-consistently from the particle flux balance). Tonks and Langmuir (T&L) found that the region of plasma arc can be mathematically split into "plasma approximation," where strict quasineutrality is assumed and "sheath approximation", dominated by the electric field. The corresponding two regions of the plasma-wall transition layer are often referred to as "the pre-sheath" and "the Debye sheath" regions. T&L found approximate solutions for these two regions in plane, cylindrical and spherical geometries with the assumption that the ions are generated at rest. This is known as the "cold" or "singular" ion source scenario, unlike the much more complex "warm" or "regular" ion source case, which was formulated by Bissell and Johnson (1987) (B&J) with the Maxwellian ion source. Unfortunately, the B&J model solution turns out to be limited to a narrow range of ion source temperatures and rather unreliable due to approximations aimed at obtaining the solution. Kos et al. (2009) and Jelić et al. (2009) have recently managed to solve the B&J model without any restriction. However, the above authors did not apply their model to various plasma purposes, e.g., for a particular task to investigate fluid and kinetic properties of the plasma-edge boundary in terms of convenient expressions of the form of Bohm criterion (Bohm, 1949), i.e., its possible kinetic generalization (Harrison and Thompson, 1959). In addition, no fluid generalization of the Bohm criterion for warm ions was proven up to date. This is a result which will emerge from the present work in a natural manner.

2. Theoretical backgrounds

The general Tonks and Langmuir (1929) problem consists in simultaneously solving Boltzmann's equation for ion VDF $f_i(x, v)$:

$$v\frac{\partial f_i}{\partial x} - \frac{e}{m_i}\frac{d\Phi}{dx}\frac{\partial f_i}{\partial v} = S_i(x, v) \qquad (1)$$

and Poisson's equation for the potential $\Phi(x)$:

$$-\frac{d^2\Phi}{dx^2} = \frac{e}{\epsilon_0}(n_i - n_e)\ , \qquad (2)$$

where the collisional source term $S_i(x, v)$ on the right-hand side is a function describing the relevant microscopic physics involved in the model of interest, with x the Cartesian space coordinate, v the particle velocity, e the positive elementary charge, m_i the ion mass, and $\Phi(x)$ the electrostatic potential at position x), and Poisson's equation for the potential, respectively, where ϵ_0 is the vacuum dielectric constant, and n_i, n_e are the ion and electron densities, respectively. We introduce the normalized quantities of interest as follows: $\frac{e\Phi}{kT_e} \to \Phi$, $\frac{m_i v^2}{2kT_e} \to v^2$, $\frac{x}{L} \to x$, $\frac{n_{i,e}}{n_{e0}} \to n_{i,e}$, $\frac{T_n}{T_e} \to T_n$, $\frac{T_i}{T_e} \to T_i$, $\frac{\sqrt{2}c_{s0} f_i}{n_{e0}} \to f_i$, $S_i L \to S_i$, where $c_{s0} \equiv \sqrt{kT_e/m_i}$ and L is any characteristic system length, (usually, the half-length of the plane-parallel discharge). Subscript i, e denotes that equation is equally applicable to ions and electrons. Eqs. (1, 2) in the normalized form reads:

$$\frac{\partial f_i}{\partial x} - \frac{d\Phi}{dx}\frac{\partial f_i}{(\partial v^2)} = \frac{S_i(x, v)}{v} \quad \text{and} \quad -\varepsilon^2\frac{d^2\Phi}{dx^2} = n_i - n_e\ , \qquad (3)$$

respectively. Here $\varepsilon \equiv \lambda_D/L$ (with the Debye length $\lambda_D = \sqrt{\epsilon_0 kT_e/n_{e0}e^2}$ and n_{e0} the electron density at the center of the plasma) is the smallness parameter of the problem. Assuming that the electron density is Boltzmann-distributed $n_e = \exp(\Phi)$, the procedure described in Ref. (Kos and Jelić, 2010) leads to the solution in the form:

$$\begin{aligned} B\int_0^1 & \exp[\Phi(x') - \Phi(x)]\exp\left[\frac{1}{2T_n}\{\Phi(x') - \Phi(x)\}\right] \\ & \times \quad \mathrm{K}_0\left\{\frac{1}{2T_n}|\Phi(x') - \Phi(x)|\right\}dx' \\ & = \quad 1 - \varepsilon^2\exp(-\Phi)\frac{d^2\Phi}{dx^2}\ , \end{aligned} \qquad (4)$$

with $B = \frac{1}{2\pi}\sqrt{\frac{m_i}{T_n m_e}}\frac{n_0}{n_{av}}\exp\left(\frac{e\Phi_w}{kT_e}\right)$, emerging from the charge flux balance (see e.g. Kos et al., 2009) at the wall with Φ_w the wall potential and n_{av} the average ion density. Since here we are interested in the "vanishing-ε" case, i.e., only in the quasineutral core plasma and its boundary, after interchanging the dependent and independent variables we obtain the B&J integral-differential equation, for unknown function $\Psi(\Phi')$:

$$\frac{1}{B} = \int \Psi(\Phi') \exp\left[\left(1 + \frac{1}{2T_n}\right)(\Phi - \Phi')\right] \times \mathrm{K}_0\left(\left|\frac{\Phi - \Phi'}{2T_n}\right|\right) d\Phi' \ . \tag{5}$$

Once function $\Psi(\Phi')$ is known, either from numerical calculations (Kos et al., 2009) or from analytic approximation (Kos et al., 2011), solution to the above equation is obtained, it is possible to calculate the ion velocity distribution, which in normalized variables in accordance to B&J reads:

$$f_i(\Phi(x), v) = B \int_{\Phi'} \Psi(\Phi') \exp(\Phi') \times \frac{\exp\left\{-[v^2 - (\Phi' - \Phi)]/T_n\right\}}{\sqrt{v^2 - (\Phi' - \Phi)}} d\Phi' \ . \tag{6}$$

Furthermore, all the moments of ion VDF, i.e. the density $n = \int f(v)dv$, directional velocity $u = \frac{1}{n}\int f(v)vdv$, and ion temperature $T = \int \frac{1}{n} f(v)(v-u)^2 dv$ and all higher moments like heat flux, energy flux etc., can be found at any location, as well as the quantity $\langle v^{-2}\rangle = \frac{1}{n}\int f(v)dv/v^2$ necessary for the calculation of the H&T plasma-sheath condition. Additional derived quantity of interest is the local polytropic coefficient function $\gamma_i(x)$ (or equivalently $\gamma_i(\Phi)$):

$$\gamma_i = 1 + \frac{n_i}{T_i}\frac{dT_i}{dn_i} \equiv 1 + \frac{n_i}{T_i}\frac{dT_i/d\Phi}{dn_i/d\Phi} \ . \tag{7}$$

Please note that although the physical properties of the last quantity, as introduced in plasma physics for the first time in Ref. (Kuhn et al., 2006), have been discussed in detail in e.g., Ref. (Kuhn et al., 2010), in the present paper one can consider its introduction just formally as a convenient abbreviation.

On the other hand, at the plasma boundary the standard procedure of expanding the charge density in the sheath $n_e - n_i$ in terms of the potential $\Phi(x)$ near the "infinitely distant" point $x_s/L \to \infty$, $\Phi_s \to 0$ (where conditions $(n_e - n_i) \to 0$ and $d\Phi/dx \to 0$ hold), yields approximation $\frac{\epsilon_0}{2}\left(\frac{d^2\Phi}{dx^2}\right)^2 = \frac{1}{2}\frac{d(n_e-n_i)}{d\Phi}\Phi^2$ from where it follows that the condition $\frac{d(n_i-n_e)}{d\Phi} \leq 0$ must hold near the sheath boundary *in the sheath, but with strict equality sign* at the sheath. Since in collisionless sheath the density gradient can be found from (non-normalized) Vlasov equations or, alternatively, from (non-normalized) systems of fluid equations, the respective pairs of equalities can be obtained in the forms:

$$\frac{dn_{i,e}}{d\Phi} = \pm\frac{e}{m_{i,e}}\int \frac{1}{v}\frac{\partial f_{i,e}}{\partial v}dv \tag{8}$$

and

$$\frac{dn_{i,e}}{d\Phi} = \mp\frac{en_{i,e}}{\gamma_{i,e}kT_{i,e} - m_{i,e}u_{i,e}^2} \ . \tag{9}$$

Therefore the marginal Bohm criterion (equality sign), in kinetic and fluid approximations, for single charged particles and single ion species plasmas, state:

$$\frac{1}{m_i}\int \frac{1}{v}\frac{\partial f_i}{\partial v}dv + \frac{1}{m_e}\int \frac{1}{v}\frac{\partial f_e}{\partial v}dv = 0 \tag{10}$$

and

$$\frac{1}{\gamma_i kT_i - m_i u_i^2} + \frac{1}{\gamma_e kT_e - m_e u_e^2} = 0 \ . \tag{11}$$

respectively, when the quasineutrality condition at the plasma sheath is employed. Assuming that the electrons are near to a perfect thermodynamic equilibrium, their density derivative in both kinetic and fluid models is $dn_e/d\Phi = -en/kT_e$. The ion derivative, however, does not depend on the model, so the generalized Bohm criteria in two models become $m_i\left(\langle v_i^{-2}\rangle\right)^{-1} = kT_e$ and $m_i u_i^2 = kT_e + \gamma_i kT_i \equiv c_s^2$ respectively, (where the term $\langle v^{-}2\rangle$ is obtained after partial integration in kinetic integrals as shown in Eqs. (8) and (9)), or in normalized variables, respectively:

$$\left(\langle v_i^{-2}\rangle\right)^{-1} = 1 \quad \text{and} \quad u_i^2 = 1 + \gamma_i T_i \equiv c_s^2 \ , \tag{12}$$

where any velocity is normalized to c_{s0} and any temperature to electron temperature T_e.

The problem arises with the kinetic quantity $\left(\langle v_i^{-2}\rangle\right)^{-1}$, which has no obvious physical meaning sense. In fact, it has been argued in Ref. (Allen, 1976) that the dispersion relation:

$$\omega_i^2 \int_0^\infty \frac{\partial f_i(v)/\partial v}{v - \omega/k}dv + \omega_e^2 \int_{-\infty}^\infty \frac{\partial f_e(v)/\partial v}{v - \omega/k}dv = k^2 \ , \tag{13}$$

(where $\omega_{i,e}^2 = n_{i,e}e^2/\epsilon_0 m_{i,e}$ represent the ion and electron plasma frequencies respectively, and ω and k are electrostatic plasma wave frequency and wave number respectively) in the limit of long wavelengths is equivalent to the marginal Harrison and Thompson criterion (1959), which is here formulated in the form of Eq. 10. Although Allen's considerations can be readily extended to the wide class of arbitrary electron velocity distributions which satisfy $\partial f_e(v)/\partial v = 0$ in the vicinity of the ion phase velocity it is, nevertheless, difficult to calculate explicitly the integrals appearing in Eq. 10 in terms of measurable observables such as the fluid velocities, temperatures and higher moments of

VDF's. There is just a single attempt towards this direction done by Riemann (1991), which was based on the Taylor expansion if integrating the function for rather "cold" ions, yielding $\gamma_i = 3$, but it has been shown in Ref. (Kuhn et al., 2006) that even in the cases where the ion temperature is negligible but *not* vanishing, the value of γ_i is considerably higher (7-8, depending on the ion production mechanism) than proposed value 3, which is reserved for ideal adiabatic processes but, strictly speaking, can be observed only for singular ion VDF's (Dirac δ-function) and "water-bag" (Davidson, 1972) VDF's.

The *essential* novelty of the present work is that γ may *not* be considered a constant (taking values 1, 5/3, or 3 for the isothermal, adiabatic flow with isotropic pressure and 3 for the one-dimensional adiabatic flow), as presented in any classic textbook on plasma physics, but, rather, a function of the position, taking at the plasma sheath a particular value to be found which, moreover, assures that the *exact equality* takes place. This means that the plasma sheath boundary, being a point of the electric field singularity, is, indeed, the "sonic"surface (Mach number equal to unity) according to Stangeby and Allen's hypothesis made in fluid theory (Stangeby and Allen, 1970) and Allen's hypothesis argued for the kinetic model (Allen, 1976), but confirmation of this requires redefining ion sound velocity c_s via employing consequently the local value of the polytropic coefficient. This task cannot be carried out for general VDF's but more readily with particular ones as appearing in various discharge scenarios, and possible further generalizations might be done only a posteriori, i.e., once all such relevant scenarios will be investigated one by one.

3. Results

Numerical and analytical solutions to Eq. (5), based on the numerical procedure developed by the authors with collaborators is shown in Fig. 2, where we show the plasma potential profiles for varying the ion source (neutral) temperatures (T_n) in a wide range, so as to yield the final temperatures (T_n) which correspond to plasmas under various conditions of practical interest. Note that with an increased neutral temperature the potential drops, as measured from the center (subscript "0" in notation bellow) to the edge of the plasma boundary (subscript "s" in notation bellow) decreases (higher T_n means a smaller Φ_s). Quantitatively, this result is shown in Fig. 3 where we show the relation between the ion source temperature and the edge potential. It is important to point out that the electric field at the plasma

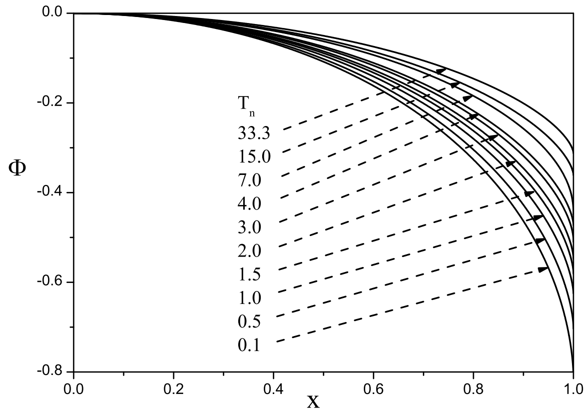

Figure 2: Potential profiles obtained for different ion source (neutral gas) temperatures.

boundary is always singular, meaning that the solution to the plasma equation breaks there, i.e., $E(\Phi_s = \infty)$. Note Fig. 3 that for the zero ion-source temperature the maximum possible potential drop $\Phi_s = -0.854...$ is obtained, which is the well-known "classical" T&L limit. The solution to Eq. (5) is just a prerequisite

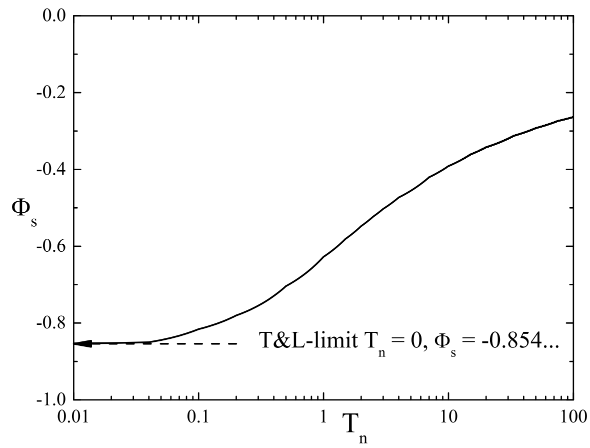

Figure 3: The plasma-edge potential Φ_s as function of the neutral temperature.

for obtaining the actual ion velocity distribution, which is essentially different from the temperature of "parent" particles, i.e., neutrals, which are supposed to be Maxwellian. The final ion velocity distribution function (VDF), as obtained from Eq. (6) is shown in Fig. 4(a) for a particular example of $T_n = 1$. It is plotted just as several places in the plasma *and* in the sheath region. It should be noted that in the case where ions are created exclusively from the ionization ("collision-free" or CF-T&L model) or the ion VDF is always characterized by

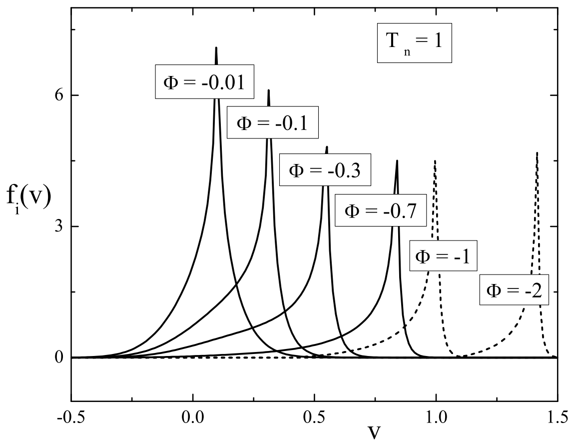

Figure 4: Ion velocity distribution functions at several positions in the discharge (dashed lines indicate VDF's in the sheath region) for a particular case of $T_n = 1$

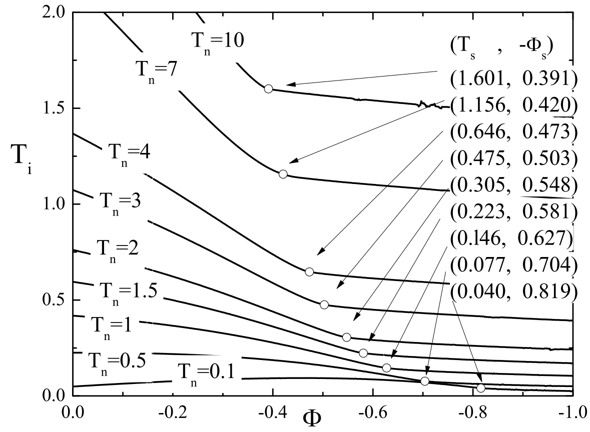

Figure 5: The ion temperature profiles as calculated from sufficiently dense sets of velocity distributions as function of potential, each curve for a particular ion source temperature.

a sharp peak, while in the case of ion source from collision dominated source ("charge-exchange" CX-T&L model) the ion VDF resembles to a Maxwellian [for cold ion sources one can see Ref. (Kuhn et al., 2006), while for warm ion-CX-source a T&L model has not been developed yet].

To obtain the profiles of fluid quantities versus x or versus $\Phi(x)$ one needs to calculate all the moments of VDF's at sufficient number of points for each particular ion source temperature. In Fig. 5 we show a family of that way calculated actual ion-temperature profiles, for various ion-source temperatures. Firstly, it is evident from this figure how the actual ion temperature behaves in B&J model. Namely, it is always much below the in source temperature, and decreases from the center towards the edge, with a more or less sharp "knee" at the plasma boundary.

The decrease of the temperature in the sheath is due to the effect of VDF's "cooling" when accelerated in a strong sheath electric field, as explained in Ref. (Kuhn et al., 2006), For the present paper, in fact, we even do not need to know plasma parameters far from the boundary. On the contrary, *at* and *near* the plasma boundary we just need an excellent resolution, i.e., high density of points at which we calculate fluid quantities, so that the derivatives of such curves (temperature and density), can be safely obtained, and this is not a trivial numerical task. Some noise might appear in derivatives that should be filtrated appropriately. Nevertheless, we obtain these quantities i.e., ion density, directional velocity, temperature, energy, and derivatives enough reliably, so that the ion polytropic coefficient

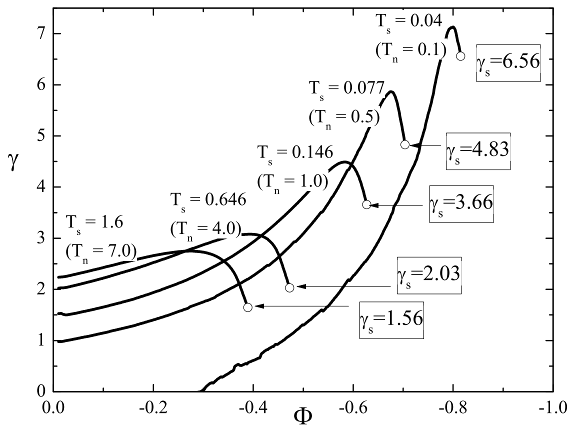

Figure 6: Ion polytropic coefficient plotted as functions of local plasma potential for various ion source temperatures.

function profiles can be calculated via Eq. (7), as illustrated in Fig. 6. We show there the $\gamma_i(\Phi)$ curves only in the plasma region for better resolving this part from the sheath region. It seems that the plasma sheath boundary coincides with the point of inflections of $\gamma_i(\Phi)$ curves, but it still has to be proved. Nevertheless, we describe the qualitatively concealed part, i.e., that $\gamma_i(\Phi)$ functions decrease slightly in the sheath and again start to increase near the sheath boundary due to adiabatic cooling, tending for very high potential drops in the sheath to $\gamma(\Phi \to -\infty) = 3$, as proven in Ref. (Kuhn et al., 2006). In Fig. 7 the dependence of γ_i on the ion temperature at the plasma boundary is shown. This figure is the first result ever presented on $\gamma_i(T_i)$. In future we

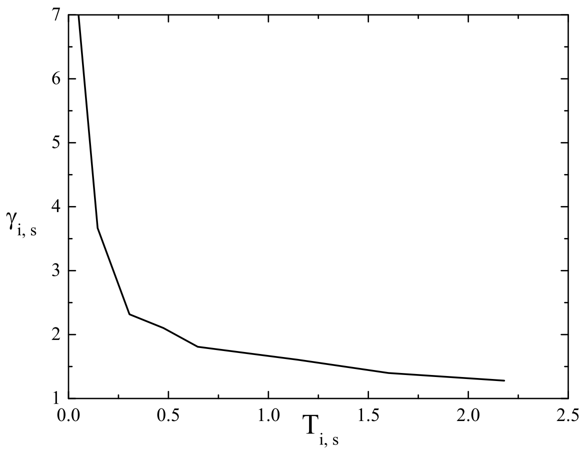

Figure 7: The dependence of $\gamma_{i,s}$ on the local ion temperature at the plasma edge-sheath boundary.

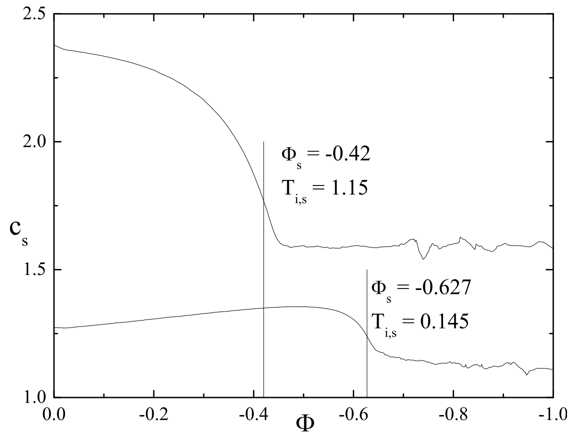

Figure 8: Profiles of ion-sound velocity according to fluid formula $c_s = \sqrt{1+\gamma_i T_i}$ obtained for two ion temperatures which attain values much lower than electron and comparable to isothermal plasma cases.

need to obtain such curves for various physical scenarios, i.e., supposed ion velocity distributions for possible generalization of fluid theory with warm ions at the sheath edge, which is at the moment completely missing in plasma physics.

We have prepared all the elements to calculate the ion-sound velocity. First, in Fig. 8(a) we show two complete profiles of ion-sound velocity according to fluid formula $c_s = \sqrt{1+\gamma_i T_i}$ obtained for two ion temperatures, which attain values much lower than electron and comparable to isothermal plasma cases, respectively. This is a brand-new result in plasma physics, where up to now the ion-sound velocity was, as a rule, calculated under assumptions of thermodynamic equilibrium with constant value $c_s = \sqrt{1+\gamma_i T_i}$ assuming T_i-constant and $\gamma_i = 1$, 3, and 5/3 by a rule. Such approach might be quantitatively acceptable for certain plasmas, but in general, it is not correct. In Fig. 9 we show the *crucial* result of comparing the ion sound velocity at the plasma sheath with the ion directional velocity there. Namely, the hypothesis of strict equality of these two velocities at the plasma-sheath boundary done by Stangeby and Allen (1970) for the fluid model and later on by Allen (1976) for the kinetic model has never been confirmed explicitly for warm-ion cases. We show here, by presenting both quantities as calculated independently from each other, that the identity $u_i = c_s$ at the plasma sheath boundary holds (within the numerical error done during determining exact values at the plasma-sheath boundary). The present approach is intrinsically a new one, identified so far only in cosmic plasmas [see e.g., Refs, Pudovkin et al. (1997); Cohen et al. (2007)], but never tackled and elaborated self-consistently before. In this particular situation, it demonstrates the necessity of employing the values of local polytropic coefficient function at the plasma sheath boundary, which is rather a sharp function of local potential, taking at the plasma sheath boundary a value which strongly depends on the ion temperature. However, for fusion plasmas, this dependence is not so critical because the value of γ_i converges somehow to a constant value 1.5, which can be used for the first trial approximation in Scrape-Off Layer fluid codes.

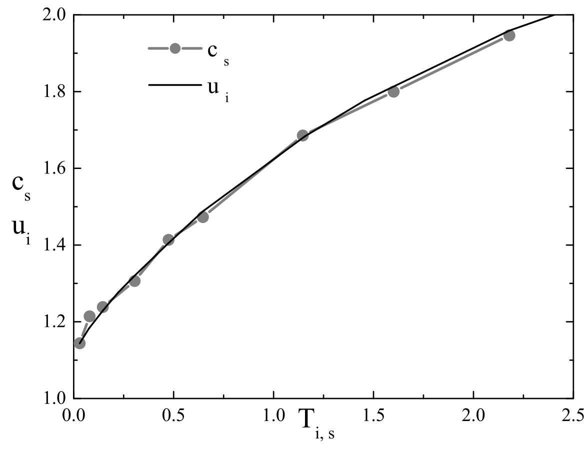

Figure 9: Comparison of ion-sound velocity with the ion directional velocity at the plasma edge as a function of the ion temperature at the plasma sheath boundary.

4. Summary and Discussion

The fluid Bohm criterion for the case of finite ion-source temperature has never been proved in fluid approach neither theoretically nor numerically. Any attempt on finding the sheath and plasma link in such plasmas was a physical oversimplification of the physical scenario in previous fluid models. This is due to fact that in the previous century, by rule, a local thermodynamic equilibrium was supposed near the plasma boundary in fluid models, even though we have been aware of the fact that such an assumption, while possibly numerically acceptable, is physically intrinsically incorrect. In order to overcome this limitation, a kinetic approach expressed in kinetic generalization of the Bohm criterion, formulated by Harrison and Thompson (1959), has been considered, but never actually employed, due to the lack of its physical practical value. Emerging quantity, $\langle v_i^{-2}\rangle^{-1/2}$ equals to *unperturbed* ion sound velocity for *negligible* ion temperature. It coincides with the directional ion velocity also in the case of perfectly "cold" ions, and, according to Riemann (1991) *can* be expressed as $\langle v_i^{-2}\rangle^{-1/2} \sim 1 + 3T_i$ for negligible ion temperatures but, otherwise, can *not* be expressed as anything meaningful. Unfortunately, this "otherwise" means *always* i.e., not only in real plasmas but also in any fairly consistent plasma-sheath model. In fact, quantity $\langle v_i^{-2}\rangle^{-1}$ has no physical meaning and no practical value. On the contrary, the fluid theory based on physical processes known in advance and a velocity distribution function obtained either from a mathematical model (like here) or from computational simulations (see e.g., Jelić et al., 2007) containing the missing link between the fluid and kinetic theory, i.e., the concept of polytropic coefficient function proposed by Kuhn et al. (2006, 2010) instead of dealing with constant values, yields results which are fully self-consistent. We have shown here that in the case of an arbitrary ion temperature, the ion sound velocity can be found provided T_i and γ_i are known at the sheath boundary. Moreover, we have shown that the ion directional velocity is identical to ion-sound velocity based on these data, and this suffices to calculate the main quantities of interest in any plasma, i.e., the particle and energy to the boundaries, *without* calculating sheath parameters. However, it should be pointed out that our results emerge from a collision-free model, and might not be applicable to collision dominated plasmas. Fortunately, the results obtained in Ref. Kuhn et al. (2006) for both CF and CX models clearly indicate that in the case of "cold" ions (born at rest) the results regarding both the ion polytropic coefficient function and the ion-sound velocity are very similar, in spite of the fact that the ion velocity distributions in these two physical scenarios are rather different in shape, and this might be similar with "warm" ion sources as well. Of course, the degree of validity of such an assumption has to be investigated in a warm-ion plasma in detail in the near future.

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